LEARNING CURVE® TRANSLATING VaR USING $\sqrt{T}$

Value-at-risk is the maximum portfolio loss expected to occur over a specified time horizon, with a specified probability. For example, the measure of VaR recommended by the **Bank for International Settlements** is the maximum loss a portfolio is expected to suffer over 10 trading days, with 99% probability. VaR can measure both short-term risk, such as a trader's risk carried on a book overnight, as well as longer term risk, such as risk carried over the quarter. It is generally believed that for portfolios with less liquid investments or investors with longer anticipated holding periods, the time horizon should be longer. One challenge is to compare or aggregate VaRs that are based on different time horizons.

For example, a bank might wish to adjust short-term and long-term VaRs from different divisions onto a comparable basis in order to allocate capital to the areas with the greatest risk-adjusted returns. For almost all portfolios, VaR increases with the length of the time horizon chosen because absolute volatility increases with time. However, the degree to which VaR changes with time can be elusive. Many organizations use the square root of time, $\sqrt{t}$, as an approximation based on the notion that VaR is simply a multiple of the portfolio standard deviation and that the standard deviation of an asset's return increases in proportion to $\sqrt{t}$. Under this translation, if the one-day VaR at 99% probability is USD1 million, then the 10-day VaR at 99% probability is USD3,162,278 (USD1 million* $\sqrt{10}$).

Under certain conditions this translation is a reasonable approximation. However, these conditions rarely hold in practice, so caution must be used. The $\sqrt{t}$ translation is exact only when the following conditions are met:

1. The underlying factors are joint normally distributed. Any linear combination of normally-distributed random variables is itself normally distributed, with a fixed relationship between standard deviations and tail probabilities. As long as the disribution of asset returns per unit of time is independent and identically distributed, the standard deviation is proportional to $\sqrt{t}$. However, normality is often violated due to asset returns that display "fat tails."

2. Constant variance matrix (i.e. constant volatility and correlations). In order for asset returns per unit of time to be independent and identically distributed, the variance matrix must be constant over time. However, the term structure of implied option volatility is generally not flat.

3. No drift of underlying factors. As the mean of a normal distribution changes, all its quantiles must change by the same amount. For example, with a 0% mean and 1% standard deviation, the 5% tail is -1.645%. If the mean is 1%, the 5% tail is -0.645%. Zero drift may be assumed for shorter time horizons, but may not be reasonable over long periods.

4. No optionality in the portfolio. Specifically, the portfolio must have zero gamma, zero time decay and a delta that is not sensitive to time. If a random variable is normally distributed, any non-linear function of that variable is not normally distributed. Thus, a non-zero gamma in any of the instruments precludes normal distribution. Zero time decay is necessary for the same reason as zero drift. And due to time value, the delta of an option changes over time even if the underlying does not. If deltas change, so do the portfolio risk weightings, and the returns of the portfolio per unit of time are not independent and identically distributed.

COMPARISON OF VaR WITH DIFFERENT TIME HORIZONS

	99%	95%
1 Day VaR	**99%**	**95%**
Analytic	0.80%	0.57%
Monte Carlo Simulation	0.77%	0.57%
10 Day VaR	**99%**	**95%**
Analytic	2.54%	1.80%
Monte Carlo Simulation	3.00%	2.51%
1 Day VaR * $\sqrt{10}$	**99%**	**95%**
Analytic	2.54%	1.80%
Monte Carlo Simulation	2.45%	1.81%

Source: Capital Market Risk Advisors

If any of the above conditions are not met, the $\sqrt{t}$ translation is only an approximation of reality. Although the conditions rarely hold, for many portfolios they will "nearly" hold, and in such cases the $\sqrt{t}$ translation may be reasonable shortcut. Many consider well-diversified global portfolios with little or no options positions to "substantially" meet these conditions. Moreover, even if they do not hold in actuality, conditions one and four are the underlying assumptions of analytic VaR (variance/covariance VaR), the least computationally-intensive and most widely used form of VaR. Thus, the $\sqrt{t}$ translation will hold approximately across time horizons for analytic VaR and will hold exactly assuming zero drift and constant variance.

For many portfolios, however, the $\sqrt{t}$ translation is unreliable. Consider VaR estimates for a hypothetical portfolio consisting of long positions in the two-year and 30-year **Treasury** strips, a long position in the **Standard & Poor's 500** futures contract and five S&P 500 option positions of varying strikes and maturities. The table at center shows the VaR of the portfolio, computed using parameters from **J.P. Morgan**'s RiskMetrics dataset, varying time horizons and probability levels, under the analytic approach and a Monte Carlo simulation. For all calculations, conditions one, two and three are assumed to hold. The analytic VaR also assumes that condition four holds. The Monte Carlo simulation, however, relaxes this assumption and instead performs a full repricing of each instrument at each trial. As the table shows, using the assumptions of analytic VaR, the $\sqrt{t}$ translation is exact. Under the Monte Carlo simulation, however, the $\sqrt{t}$ translation produces large errors.

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