

VaR for Mortgage Backed Securities

Capital Market Risk Advisors



VaR for MBS Portfolios

- ◆ *What is VaR?*
- ◆ *Why is VaR Important?*
- ◆ *How is VaR Calculated?*
- ◆ *VaR Choices*
- ◆ *VaR for MBS*
- ◆ *Common VaR Methods*
- ◆ *Which VaR is Right for You?*

What is VaR?

Value at Risk (VaR) is a mathematical approach for estimating the maximum potential loss of a given portfolio within some period of time with some likelihood of occurrence.

Probability Distribution of Portfolio Value



Why is VaR Important?

As part of a larger risk management framework, Value at Risk provides insight into the risks being taken.

- ◆ Identifies risk concentrations across dimensions
 - trader
 - market
 - product

- ◆ Provides an estimate of a maximum expected portfolio loss over a various time periods
 - daily
 - weekly
 - annual

- ◆ Quantifies the risk reduction due to diversification

- ◆ Provides an aggregated view of different risk types

What VaR is Not

Value-at-Risk does not provide solutions for all of risk management's needs.

- ◆ Does not select “good” and “bad” investments
- ◆ Provides very limited information on liquidity and operational risks
- ◆ Does not capture event risks or severe market shocks
- ◆ Is not a “worst case” scenario
- ◆ Is only one of many risk management tools

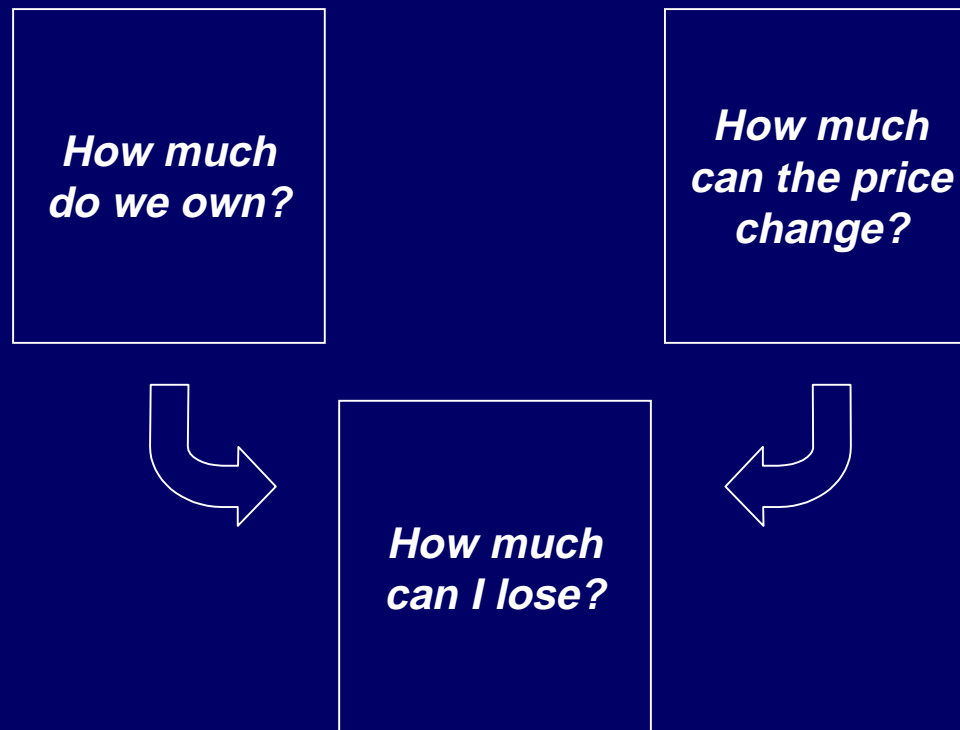
VaR is Part of IRO

Value-at-Risk should be combined with other risk management tools as part of Independent Risk Oversight.

- ◆ Scenario Analysis
- ◆ Stress Testing
- ◆ Limits Structure and Approval Process
- ◆ Empirical Backtesting & Model Calibration
- ◆ A healthy dose of common sense

Measuring Potential Loss

Value at Risk is attempting to measure the potential loss associated with adverse market movements.



How is VaR Calculated?

Value at Risk models estimate potential losses using two distinct approaches.

- ◆ Sensitivity Estimate Models - use sensitivity factors such as duration to estimate the change in value of the portfolio to changes in market rates and prices.
- ◆ Full Revaluation Models - use pricing algorithms such as bond formulae or option pricing models to estimate the change in value of the portfolio to changes in market rates and prices.

Daily VaR for One Asset

Sensitivity Approach

Product	Position Size	Sensitivity	Daily Volatility	1 Day VaR
UST 10 Year Note 5.5%	\$10MM	\$7,000/bp	10 basis points	\$70,000

How much do we own? = \$10mm

How sensitive is the position to a -1bp change in rates? = \$7,000

How much are rates likely to change with 84% confidence? = 10 bps/day

$VaR = \$7,000/bp * 10 \text{ bps} = \$70,000$

VaR for Two Assets

Sensitivity Approach

Product	Position Size	Sensitivity	Daily Volatility	1 Day VaR
UST 10 Year Note 5.5%	\$10MM	\$7,000/bp	10 basis points	\$70,000
USD 5 Year Swap 6%	\$20MM	\$8,000/bp	13 basis points	\$104,000
TOTAL	\$30MM	\$15,00/bp		\$163,000

Assume that the 10 Yr. Treasury and 5 Yr. Swap rates are 75% correlated.
Example is for illustration purposes only - results are approximated.



VaR for Two Assets Revaluation Approach

Product	Position Size	Current Price	New Price	1 Day VaR
UST 10 Year Note 5.5%	\$10MM	100 (YTM = 5.5%)	99.31 (YTM = 5.6%)	\$69,000
USD 5 Year Swap 6%	\$20MM	100 (YTM = 6.0%)	99.48 (YTM = 6.13%)	\$103,500
TOTAL	\$30MM			\$162,000

Assume that the 10 Yr. Treasury and 5 Yr. Swap rates are 75% correlated.
Example is for illustration purposes only - results are approximated.



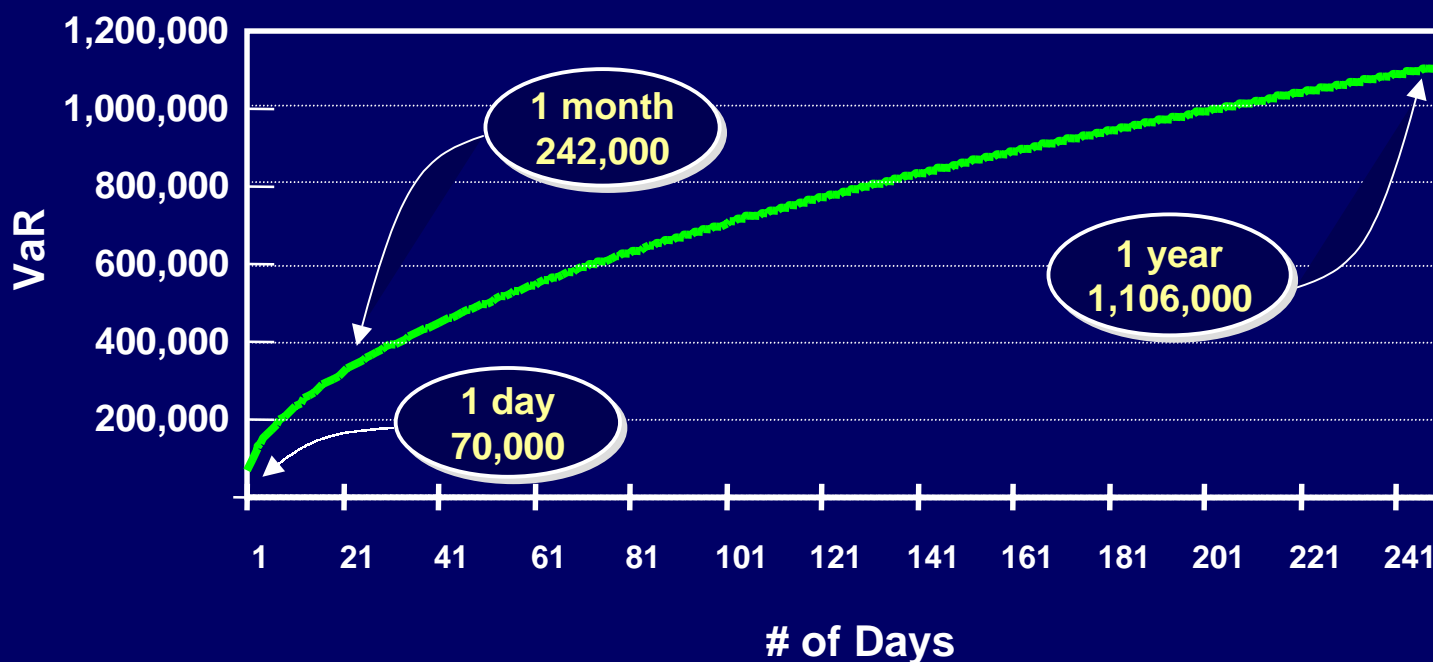
Correlation and Diversification

As correlation among assets decreases, the risk of the portfolio decreases.

Correlation	Portfolio Risk	% Risk Reduction
1.00	\$174,000	0%
.75	163,000	6%
.50	151,000	16%
.25	139,000	20%
0	125,000	28%
-.25	109,000	37%
-.50	91,000	47%
-.75	69,000	60%
-1.00	34,000	80%

Various Holding Period VaR

VaR for Different Holding Periods



One Day VaR = \$70,000

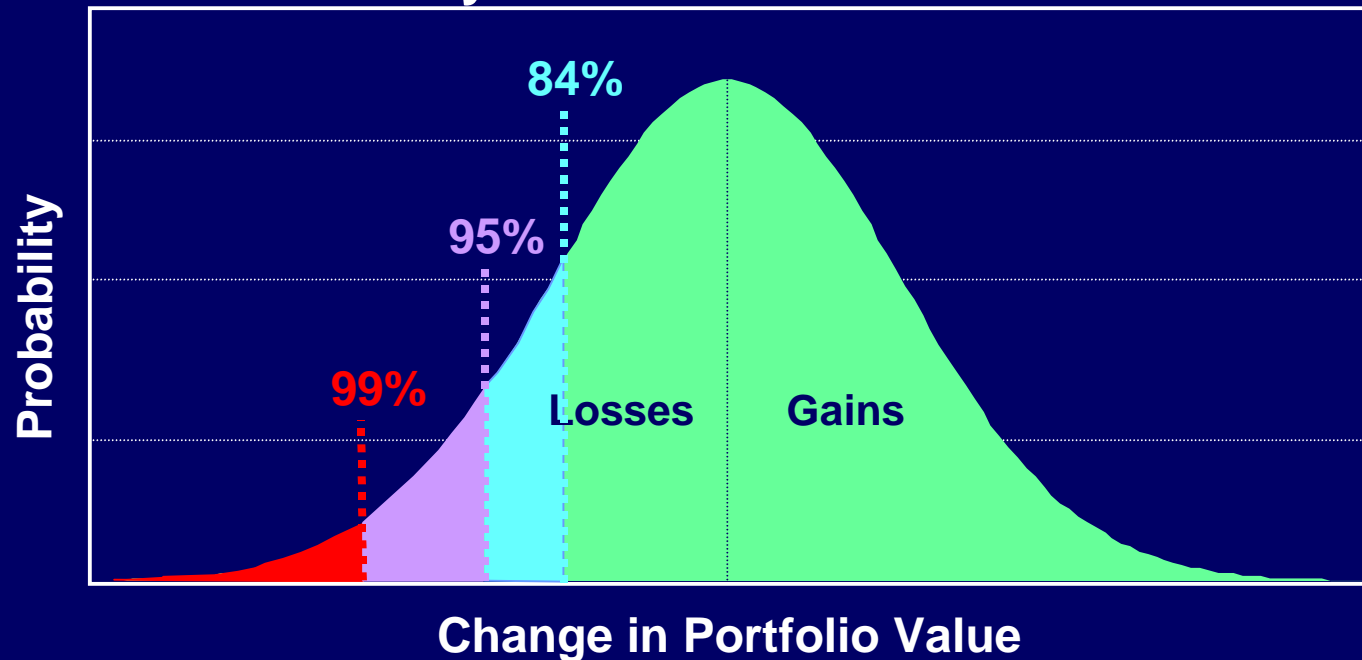
One Month VaR = $\$70,000 * \sqrt{21} \approx \$242,000$

One Year VaR = $\$70,000 * \sqrt{250} \approx \$1,106,000$



VaR Confidence Levels

Probability Distribution of Portfolio Value



$$84\% \text{ VaR} = \$70,000$$

$$95\% \text{ VaR} = \$70,000 * 1.65 \approx \$115,000$$

$$99\% \text{ VaR} = \$70,000 * 2.33 \approx \$163,000$$

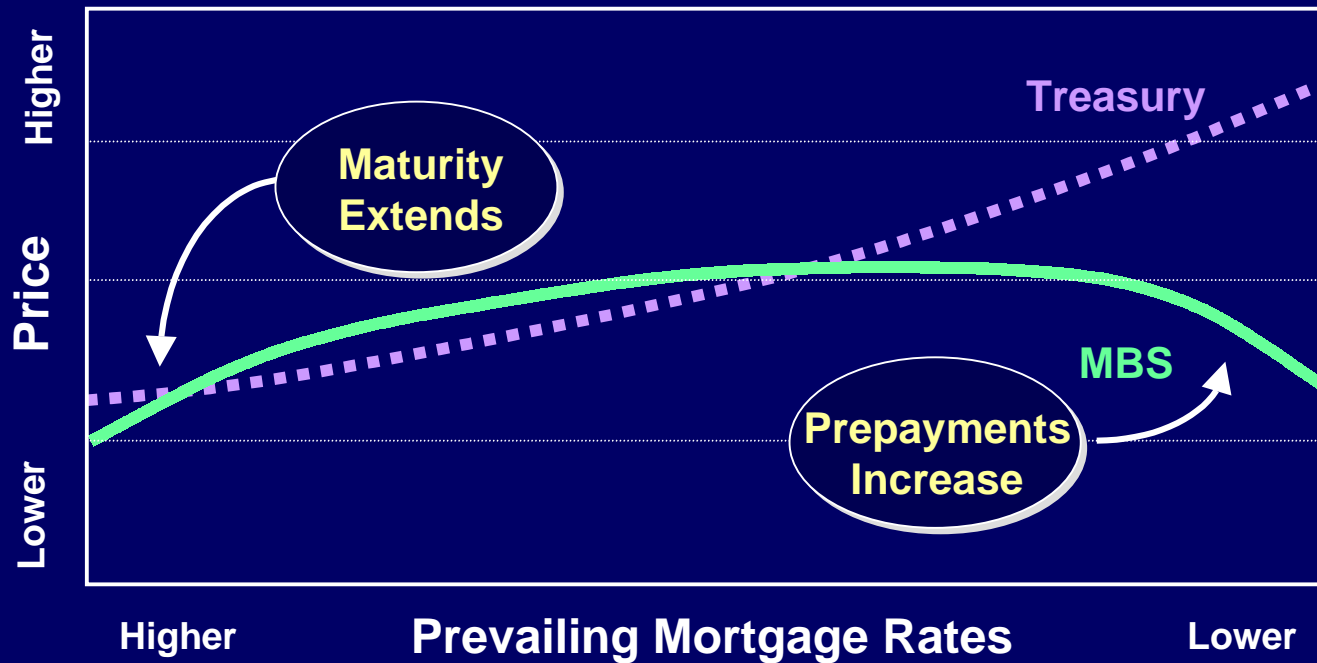
MBS Complexities

MBS contain a prepayment option that makes their price function more complex than simple Treasury Notes.

- ◆ When rates increase, fewer prepayments occur
 - ↑ a greater number of interest payments are made
 - ↓ principal is paid off over a longer period of time
 - ↓ the present value of all expected payments decreases
- ◆ When rates decrease, more prepayments occur
 - ↓ fewer interest payments are made
 - ↑ principal is paid off over a shorter period of time
 - ↑ the present value of all expected payments increases

Treasuries and MBS Payoff

Value of Treasury Note & MBS



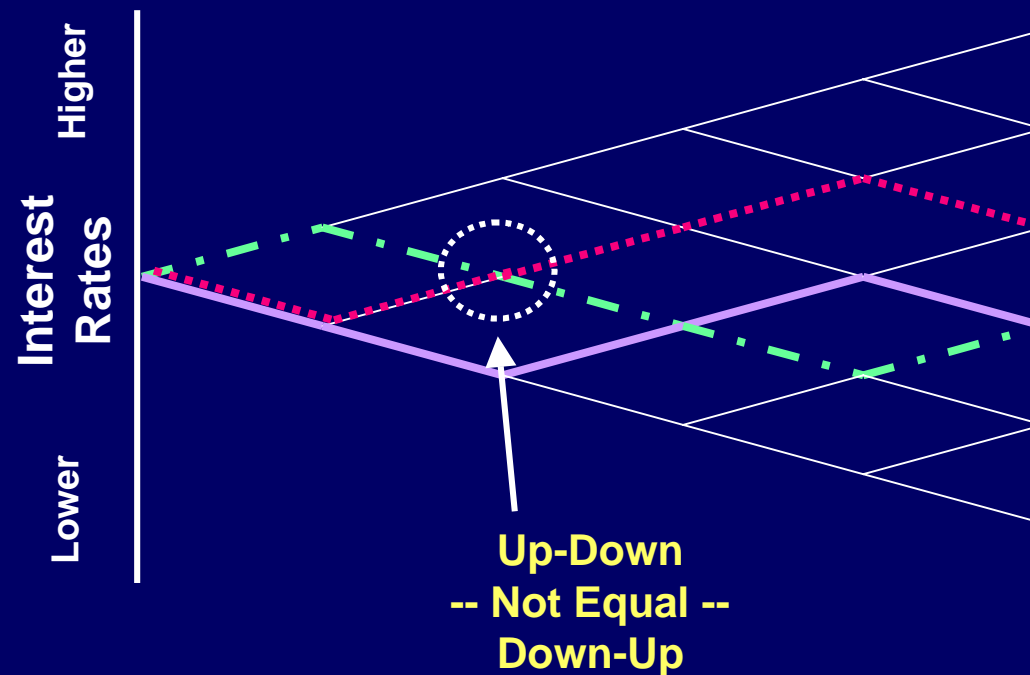
MBS Pricing Approach

Calculating the Price of a MBS requires a simulation approach.

- ◆ Generate yield curves three months forward
- ◆ Estimate the prepayment pattern given the new rate environment
- ◆ Calculate the remaining principal balance
- ◆ Generate yield curves three more months forward
- ◆ Estimate the prepayment pattern given the new rate environment
- ◆ ...
- ◆ Find the PV the expected cash flows for each possible interest rate path

Path Dependent

An initial decrease in rates followed by an increase in rates does not result in the same value as an initial increase in rates followed by a decrease in rates.



Full Revaluation Time Requirements

Applying a Full Revaluation VaR approach would require a simulation within a simulation.

- ◆ A few firms have invested heavily to achieve Full Revaluation MBS VaR
 - Super Computers
 - Massive Multiple Parallel Processing
 - Complex Run-Time Reduction Algorithms
- ◆ Sensitivity VaR provides a more practical (and less expensive) approach

MBS VaR with Convexity

Sensitivity Inputs

A Sensitivity VaR approach requires inputs for the Delta (Duration) and Gamma (Convexity) sensitivities of each position or portfolio of positions.

- ◆ Delta - Change in value for a 1 basis point change in interest rates
 - can be expressed with respect to a single yield (YTM)
 - can be expressed with respect to multiple points on the Yield Curve (Eurodollars, Swaps, Treasuries)
- ◆ Gamma - Change in value for an x basis point change in interest rates that is attributable to the option component of the MBS.

MBS VaR with Convexity Sensitivity Approach

Sample MBS Position

Sensitivity	Position Size	Sensitivity	Daily Volatility	1 Day VaR
Delta Risk (Duration)	\$10MM	\$7,000/bp	10 basis points	\$70,000
Gamma Risk (Convexity)	\$10MM	-\$400 for 10 bps	10 basis points	-\$2,000
TOTAL	\$30MM			\$72,000

Common VaR Models

There are three common approaches for calculating VaR.

- ◆ **Variance Covariance** - Fast, analytic method of calculating VaR. Works well for less complex portfolios with limited option positions.
- ◆ **Historic Simulation** - Estimates changes in value of the portfolio using actual historic market price changes. Makes no assumptions about distribution properties. Requires large amounts of historic data and is more computationally intensive than Variance Covariance methods.
- ◆ **Monte Carlo Simulation** - Generates market scenarios based on historic estimates of volatility and correlation. Estimates the change in value of the portfolio for each generated scenario. Most computationally intensive.

Variance Covariance

Variance Covariance calculates the potential portfolio loss using an historic estimate of volatility and correlation.

$$VaR = \sum_{i=1}^N \text{delta}_i^2 \sigma_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^N \sum_{j=1}^N \text{delta}_i \text{delta}_j \sigma_i \sigma_j \rho_{i,j}$$

$$163,000 \approx \sqrt{7,000^2 * 10^2 + 8,000^2 * 13^2 + 2(7,000 * 8,000 * 10 * 13 * .75)}$$

Variance Covariance

Product	Position Size	Sensitivity	Daily Volatility	1 Day VaR
UST 10 Year Note 5.5%	\$10MM	\$7,000/bp	10 basis points	\$70,000
USD 5 Year Swap 6%	\$20MM	\$8,000/bp	13 basis points	\$104,000
TOTAL	\$30MM	\$15,00/bp		\$163,000

$$163,000 \approx \sqrt{7,000^2 * 10^2 + 8,000^2 * 13^2 + 2(7,000 * 8,000 * 10 * 13 * .75)}$$

Variance Covariance

Probability Distribution of Portfolio Value

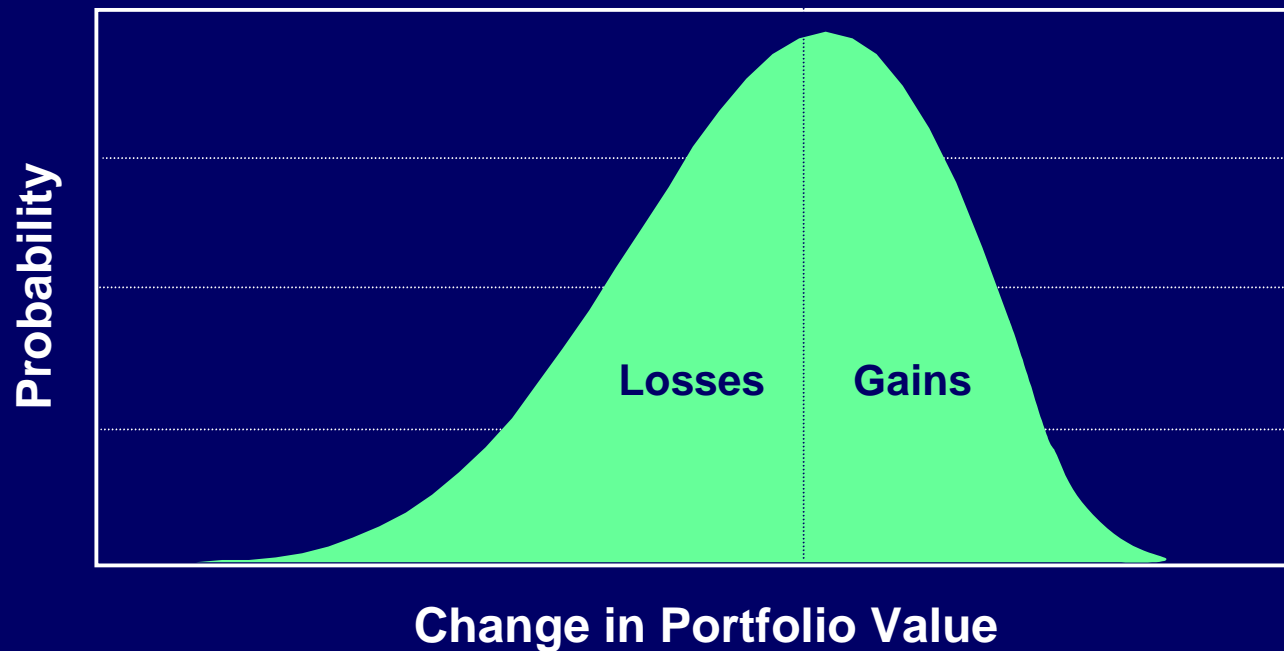


$$163,000 \approx \sqrt{7,000^2 * 10^2 + 8,000^2 * 13^2 + 2(7,000 * 8,000 * 10 * 13 * .75)}$$

Variance Covariance Option Asymmetry

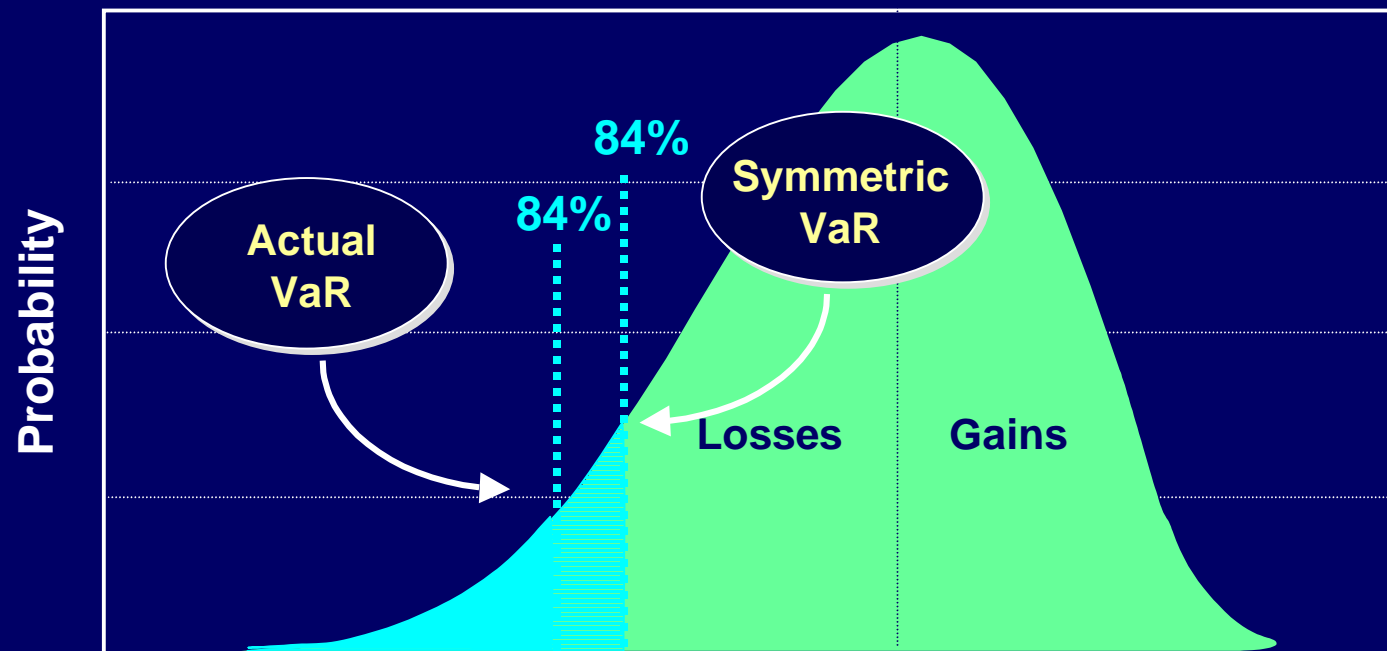
*Selling Options creates the potential for very large losses
and skews the Value Distribution.*

Long 3 Shares Stock + Short 1 Put Option



Variance Covariance Skewness Example

Long 3 Shares Stock + Short 1 Call Option



Variance Covariance Skewness Adjustment

The skewness factor determines the necessary adjustment to the symmetric VaR results.

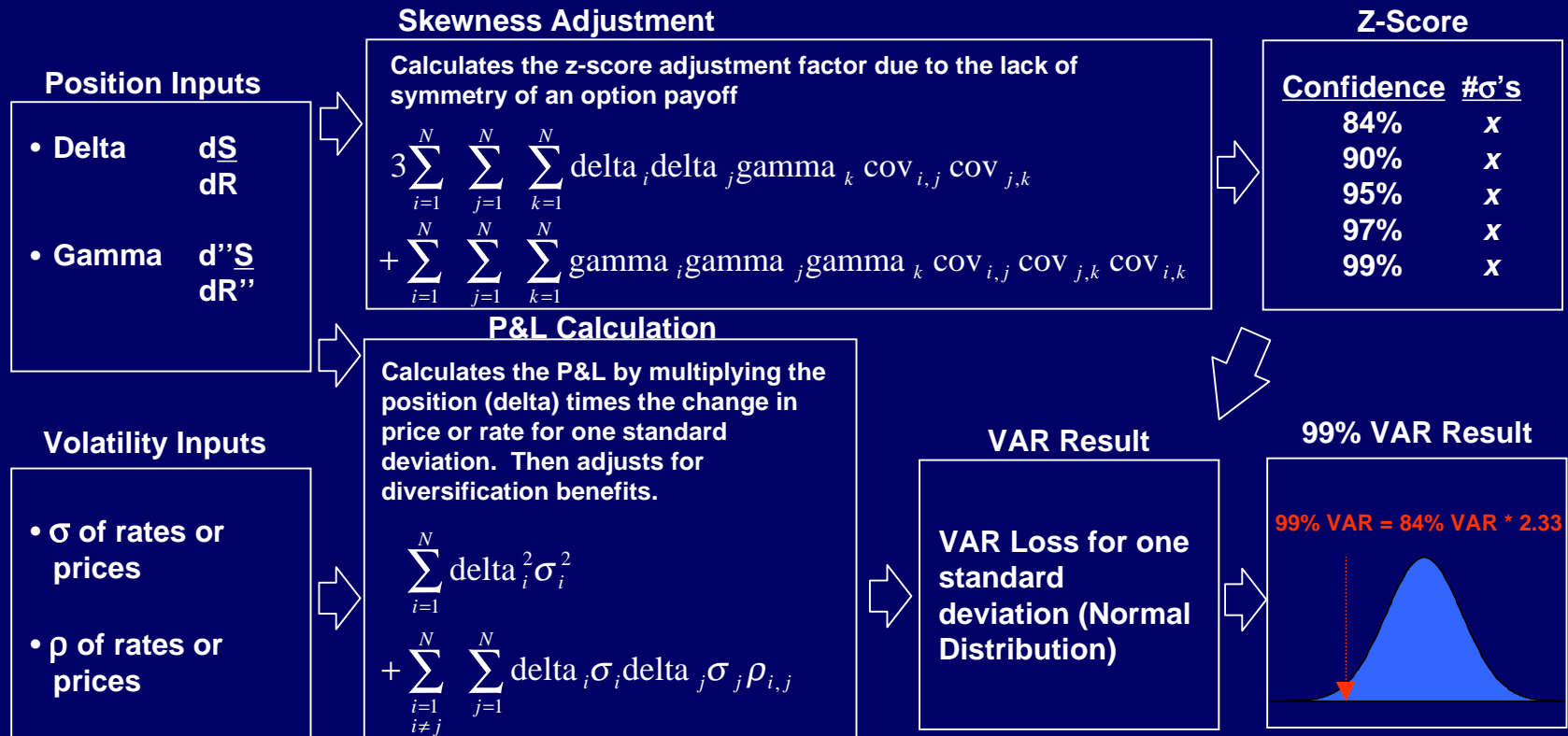
$$3 \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \text{delta}_i \text{delta}_j \text{gamma}_k \text{COV}_{i,j} \text{COV}_{j,k}$$
$$+ \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \text{gamma}_i \text{gamma}_j \text{gamma}_k \text{COV}_{i,j} \text{COV}_{j,k} \text{COV}_{i,k}$$

Variance Covariance Weaknesses for Options

However, the Variance Covariance approach cannot provide a complete P&L picture for options.

- ◆ Large changes in option price
- ◆ Points of inflection (gamma changes from + to -)
- ◆ Compound options
- ◆ Discontinuous options
- ◆ Digital options

Variance Covariance



Variance Covariance Summary

The Variance Covariance has many advantages and disadvantages:

Advantages	Disadvantages
<ul style="list-style-type: none">• Fast• Relatively easy to implement• Works well for linear risks (delta & vega)• Captures some measure of options risk (gamma)• Data sets are readily available	<ul style="list-style-type: none">• Does not revalue positions• Cannot account for complex or discontinuous payoffs• Poor at modeling the convexity/gamma effect in MBS• Cannot incorporate multiple time horizons• Assumes normal or normal-like distributions

Historic Simulation

Historic Simulation relies on time series of actual price changes - it does not assume a normal distribution.

<u>Date</u>	<u>Rate</u>	<u>Change</u>
Jan2	6.25%	.10
Jan3	6.35%	.10
Jan4	6.40%	.05
Jan5	6.15%	-.25
.	.	.
.	.	.
.	.	.
Dec31	6.20%	-.10

Historic Simulation

Sample Position from Previous Example

Product	Position Size	Sensitivity
UST 10 Year Note 5.5%	\$10MM	\$7,000/bp

Historic Simulation Calculating P&L

Historic Simulation calculates the potential portfolio loss using historic market prices.

<u>Date</u>	<u>Price</u>	<u>Price</u>	<u>Rate</u>	<u>Change</u>
Jan2	100	99	6.25%	.12
Jan3	101	97	6.35%	.10
Jan4	102	99	6.40%	.05
Jan5	101	98	6.15%	-.25
.
.
.
Dec31	100	94	6.20%	-.13

$P\&L = \$7,000 * 10bps = \$70,000$



Historic Simulation Calculating P&L

P&L is calculated for each day contained in the time series...

<u>Date</u>	<u>Price</u>	<u>Price</u>	<u>Rate</u>	<u>Change</u>		<u>P&L</u>
Jan2	100	99	6.25%	.12▶	-8,400
Jan3	101	97	6.37%	.10▶	-7,000
Jan4	102	99	6.40%	.05▶	-3,500
Jan5	101	98	6.15%	-.25▶	+17,500
.
.
.
Dec31	100	94	6.20%	-.13▶	+9,100

Historic Simulation Calculating P&L

... and then rank ordered from largest gain to largest loss.

<u>Date</u>	<u>Price</u>	<u>Price</u>	<u>Rate</u>	<u>Change</u>		<u>P&L</u>	<u>Ordered P&L</u>
Jan2	100	99	6.25%	.10>	-70,000	+175,000
Jan3	101	97	6.37%	.12>	-84,000	+91,000
Jan4	102	99	6.40%	.05>	-35,000	-35,000
Jan5	101	98	6.15%	-.25>	+175,000	.
.
.
.	-70,000
Dec31	100	94	6.20%	-.13>	+91,000	-84,000



Historic Simulation Calculating P&L

The x% confidence VaR is then found by counting down the appropriate number of P&L observations.

<u>Date</u>	<u>Observation</u>	<u>P&L</u>	
Jan 13	1	+229,000	
Mar 4	2	+217,500	
Jun 30	3	+215,300	
Oct 20	4	+199,100	
...	
Jul 7	950	-125,000	← 95% VaR
...	
Mar 12	990	-180,000	← 99% VaR
...	
Apr 19	998	-221,000	
Apr 2	999	-239,000	
Dec 21	1,000	-308,400	

Historic Simulation Options P&L

VaR for positions with options uses a two part P&L formula.

$$P \ \& \ L = \text{delta} (r_t - r_{t-1}) + \frac{1}{2} \text{gamma}_i (r_t - r_{t-1})^2$$

The first term multiplies the position by the change in price/rate. This calculates the P&L of the positions for the delta risk only. For assets that are not interest rate sensitive and are not options, this is the total P&L.

The second term then adds or subtracts an amount to adjust for the convexity or gamma effect. If the portfolio is long options, the second term reduces loss/increases the gain. If the portfolio short options, the second term increases the loss/reduces the gain.



Historic Simulation Taylor Series Expansion

$$P \ \& \ L = \text{delta} (r_t - r_{t-1}) + \frac{1}{2} \text{gamma}_i (r_t - r_{t-1})^2$$

$$-\$72,000 = -\$7,000 * 10\text{bps} + 1/2 * -400 / 10 * 10^2$$

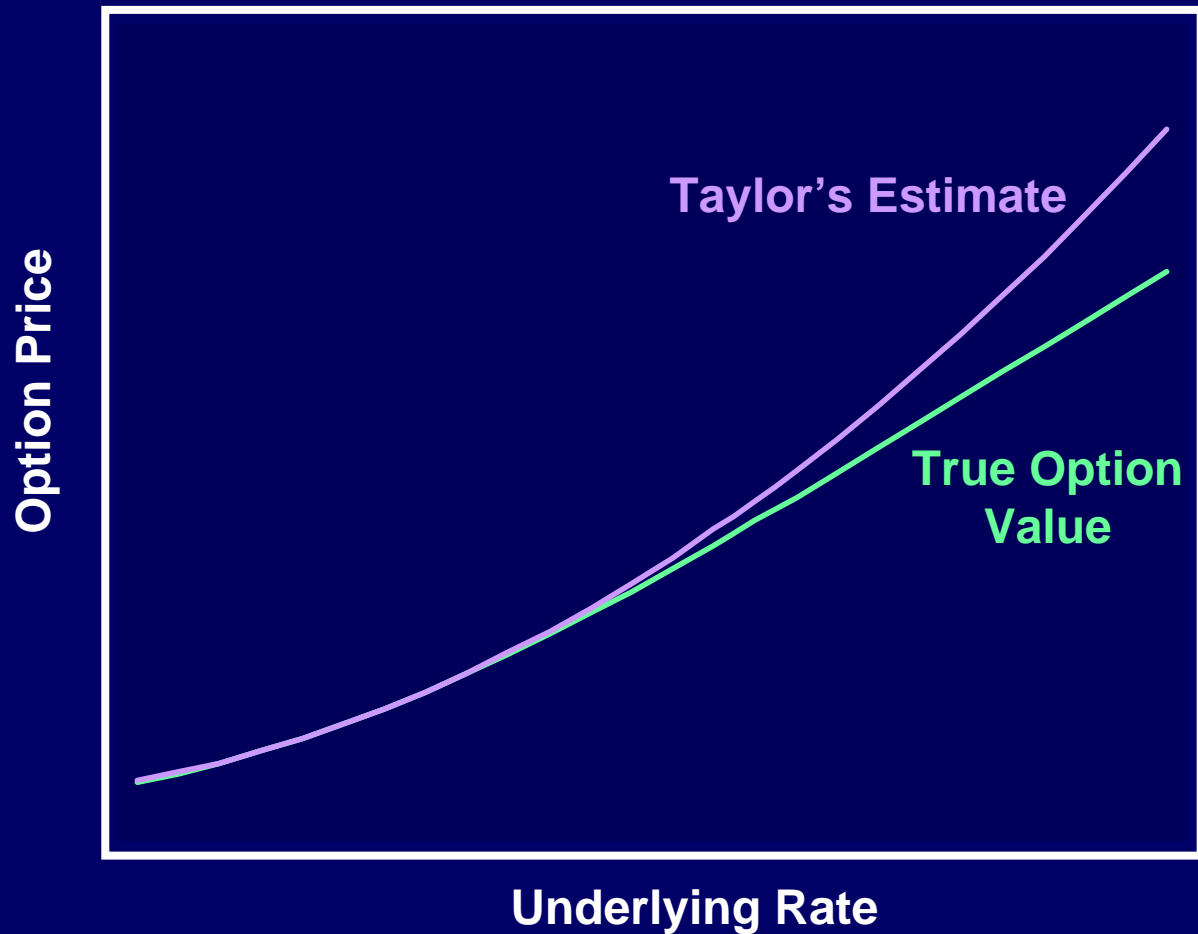
Historic Simulation

Sample MBS Position

Sensitivity	Position Size	Sensitivity	Daily Volatility	1 Day VaR
Delta Risk (Duration)	\$10MM	\$7,000/bp	10 basis points	\$70,000
Gamma Risk (Convexity)	\$10MM	-\$400 for 10 bps	10 basis points	\$2,000
TOTAL				\$72,000

$$-\$72,000 = -\$7,000 * 10\text{bps} + 1/2 * -400 / 10 * 10^2$$

Historic Simulation Estimating Gamma



Multiple Gamma Inputs

The accuracy of the Taylor's estimate can be improved by using more than one estimate of gamma.

Change in Underlying Rate

Value of 01

	-75	-25	-10	0	+10	+25	+75
Value of 01	\$4,000	\$5,800	\$6,600	\$7,000	\$7,300	\$7,500	\$7,600

Historic Simulation Taylor Series Expansion

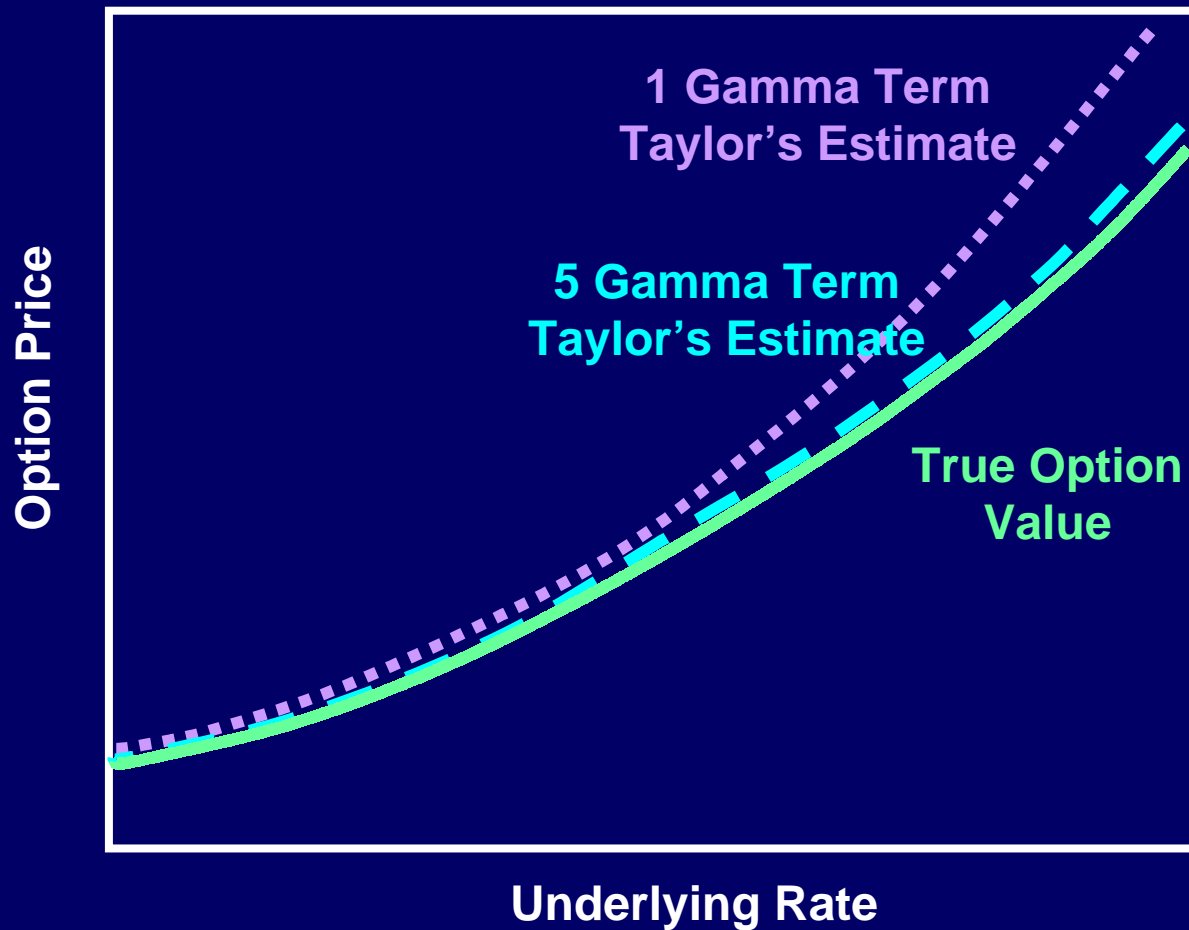
$$P \ \& \ L = \text{delta} \ (r_t - r_{t-1}) + \frac{1}{2} \text{gamma}_i \ (r_t - r_{t-1})^2$$

$$\$72,000 = 7,000 * -10\text{bps} + 1/2 * -400 / 10 * 10^2$$

$$\$187,500 = 7,000 * -25\text{bps} + 1/2 * -1,200 / 25 * 25^2$$

$$\$637,500 = 7,000 * -75\text{bps} + 1/2 * -3,000 / 75 * 75^2$$

Historic Simulation Multiple Gamma Inputs



Historic Simulation

Position Inputs

• Delta dS
 dR

Time Series Inputs

Date	Price	Price	Rate
Jan2	100	99	6.25%
Jan3	101	97	6.35%
Jan4	102	99	6.40%
Jan5	101	98	6.15%
.	.	.	.
Dec31	100	94	6.20%

P&L Calculation

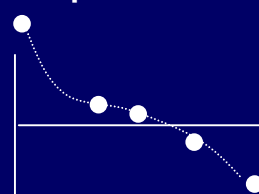
Calculates the P&L by multiplying the position (delta) times the actual change in price or rate for day in history. The second term multiplies the one half gamma position times the change in position squared. This is the addition/subtraction for the "option effect".

$$\sum_{t=1}^T \text{delta} (P_t - P_{t-1}) + \sum_{t=1}^T \frac{1}{2} \text{gamma}_p (P_t - P_{t-1})^2$$

The gamma you apply depends on the size of the price/ rate shock

Gamma Curve

Spline Fit



Position Inputs

Gamma Vector

Shock	Gamma
+100	10
+25	2
0	1
-25	-2
-100	-5

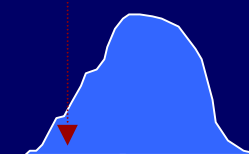
VAR Results

Sorted P&L

Date	P&L
Mar2	250
Jun15	130
Apr4	19
Apr19	-125
.	.
Nov30	-353

99% VAR Result

99% VAR = 99% Percentile Result



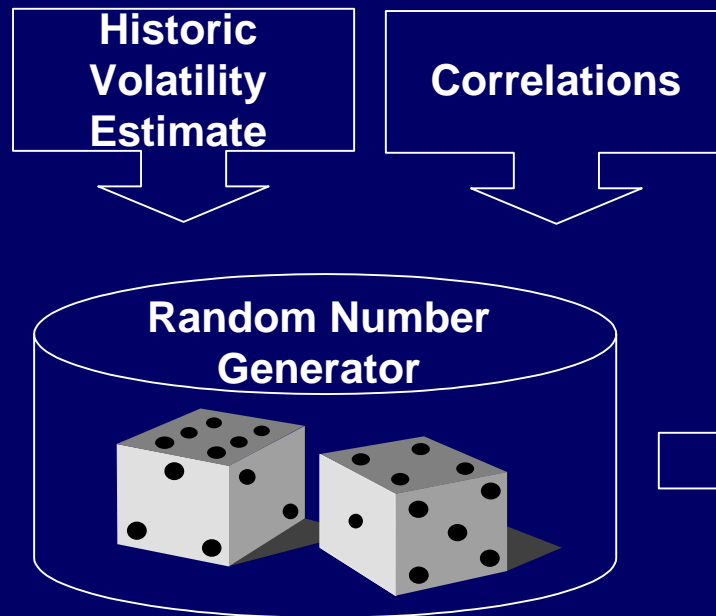
Historic Simulation Summary

The Historic Simulation using a delta gamma sensitivity approach has many advantages and disadvantages:

Advantages	Disadvantages
<ul style="list-style-type: none">• Moderately fast• Makes no assumption about distributions• Relies on volatility and correlation embedded in time series• Captures majority of options risk	<ul style="list-style-type: none">• Does not revalue positions• Can only estimate complex or discontinuous payoffs• Historic data intensive

Monte Carlo Simulation

Monte Carlo Simulation calculates the potential portfolio loss using randomly created scenarios.



<u>Scenario</u>	<u>Rates</u>
1	6.25
2	6.34
3	6.54
4	6.43
.	.
.	.
.	.
10,000	7.01

Monte Carlo Simulation

Monte Carlo Simulation calculates the potential portfolio loss using market scenarios that were created from historic volatility and correlation estimates.

<u>Scenario</u>	<u>Rate</u>	<u>Change</u>
1	6.25	.09
2	6.35	.10
3	6.54	.2
4	6.43	-.11
.	.	.
.	.	.
.	.	.
10,000	7.01	-.08


$$\text{P\&L} = \$70,000 * 10\text{bps} = 70,000$$

Historic Simulation Calculating P&L

Just like Historic Simulation, P&L is calculated for each scenario and rank ordered from largest gain to largest loss.

<u>Scenario</u>	<u>rate #1</u>	<u>change</u>		<u>P&L</u>	<u>Ordered P&L</u>
1	6.25	.09▶	-63,000	+91,000
2	6.35	.10▶	-70,000	+77,000
3	6.54	.20▶	-140,000	-63,000
4	6.43	-.11▶	+77,000	-70,000
.
.
.
10,000	7.01	-.08▶	+91,000	-140,000



Monte Carlo Simulation

Calculating P&L

The x% confidence VaR is then found by counting down the appropriate number of P&L observations.

<u>Observation</u>	<u>P&L</u>	
1	+229,000	
2	+217,500	
3	+215,300	
4	+199,100	
...	...	
950	-125,000	← 95% VaR
...	...	
990	-180,000	← 99% VaR
...	...	
998	-221,000	
999	-239,000	
1,000	-308,400	

Monte Carlo Simulation with Options

For option P&L, Monte Carlo Simulation relies on the same two part P&L formula as Historic Simulation.

$$P \ \& \ L = \text{delta} \ (r_t - r_{t-1}) + \frac{1}{2} \text{gamma}_i \ (r_t - r_{t-1})^2$$

The first term multiplies the position by the change in price/rate. This calculates the P&L of the positions for the delta risk only. For assets that are not interest rate sensitive and are not options, this is the total P&L.

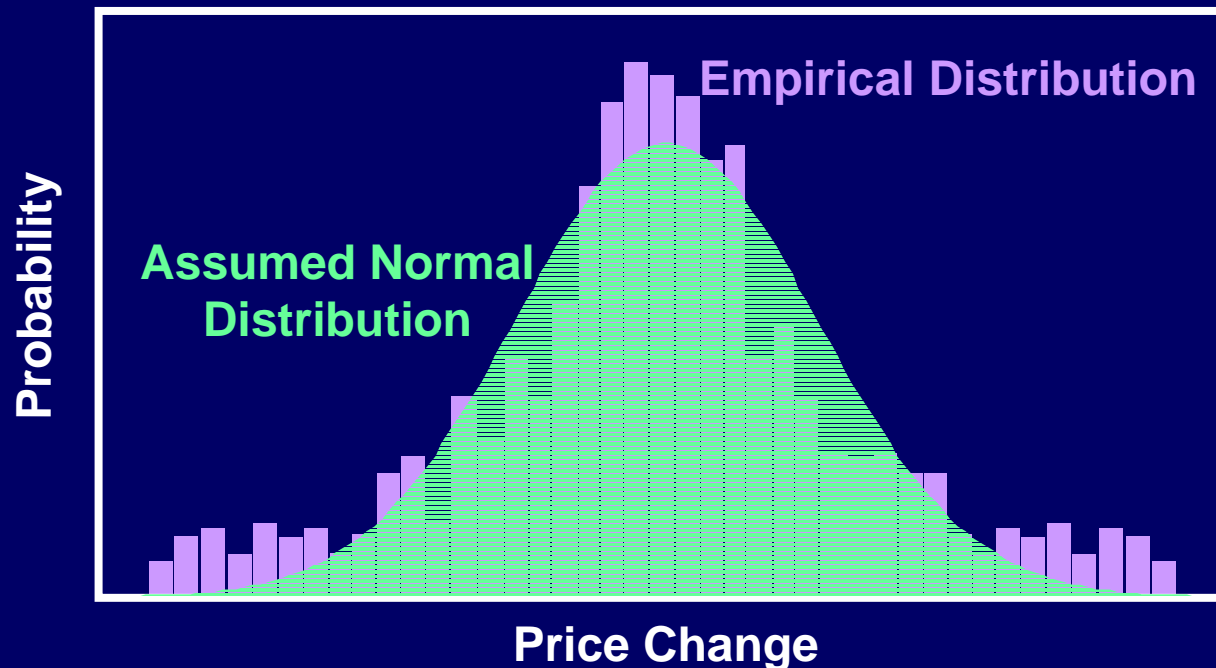
The second term then adds or subtracts an amount to adjust for the convexity or gamma effect. If the portfolio is long options, the second term reduces loss/increases the gain and thus the risk. If the portfolio short options, the second term increases the loss/reduces the gain and risk goes up.



Historic vs. Monte Carlo

The main difference between the two approaches is the shape of the price change distribution.

Probability Distribution of Price Changes



Monte Carlo Simulation

Position Inputs

- Delta dS
 dR
- Multiple $d''S$
Gammas dR''

Volatility Inputs

- σ of rates or prices
- ρ of rates or prices

Scenario Generator

Scenario	Price	Price	Rate
1	100	99	6.25%
2	101	97	6.23%
3	102	99	6.65%
4	101	98	6.15%
.	.	.	.
.	.	.	.
9,998	100	94	7.25%
9,999	100	94	7.25%
10,000	100	94	7.25%

P&L Calculation

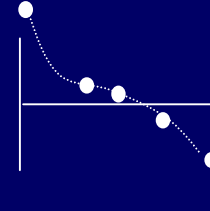
Calculates the P&L by multiplying the position (delta) times the actual change in price or rate for day in history. The second term multiplies the one half gamma position times the change in position squared. This is the addition/subtraction for the "option effect".

$$\sum_{t=1}^T \text{delta}(P_t - P_{t-1}) + \sum_{t=1}^T \frac{1}{2} \text{gamma}_p (P_t - P_{t-1})^2$$

The gamma you apply depends on the size of the price/ rate shock

Gamma Curve

Spline Fit



Position Inputs

Gamma Vector

Shock	Gamma
+100	10
+25	2
0	1
-25	-2
-100	-5

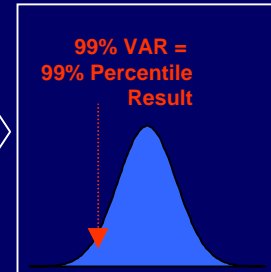
VAR Results

Sorted P&L

Date	P&L
Mar2	250
Jun15	130
Apr4	19
Apr19	-125
.	.
.	.
Nov30	-353

99% VAR Result

99% VAR =
99% Percentile
Result



Monte Carlo Simulation Summary

The Monte Carlo Simulation has many advantages and disadvantages:

Advantages	Disadvantages
<ul style="list-style-type: none">• Can approximate very complex and discontinuous payoffs• Flexible• Can incorporate multiple time periods• Captures majority of options risk	<ul style="list-style-type: none">• Moderately computationally intensive• Requires maintenance of pricing model library• Trade Position Data intensive

Selecting a VaR Approach

	Variance Covariance	Historic Simulation	Monte Carlo Simulation
Speed	Very Fast	Moderate	Slowest
Captures "Fat Tails" Effect	No	Yes	No
Historic Data Requirements	Low	High	Low
Ease of Implementation	Easiest	Moderate	Most Difficult
Reliance on Normal Distribution Assumption	High	None	Moderate
Ability to Represent Options	Lowest	Very Good	Very Good

Sample Risk Report

	Interest Rates				Foreign Exchange				Equities				Daily VaR 87%
	USD	JPY	EUR	Other	Net USD	JPY	EUR	Other	USD	EUR	JPY	Other	
Positioning													
Trader A													
Trader B													
Trader C													
Hedge/Overlay													
Trader A													
Trader B													
Trader C													
Statistic Model													
Strategy A													
Strategy B													
Strategy B													
TOTAL	0	0	0	0	0	0	0	0	0	0	0	0	
Kosovo Expansion													
Korea Conflict													
Russia Coupe													
Latin America Deval													
Japan Collapse													
Prices/Rates +50%													
Prices/Rates +20%													
Prices/Rates -25%													
Prices/Rates -50%													
Correlations = +1													
Correlations = + 50%													
Correlations = - 50%													
Correlations = -1													

